

NONSTATIONARY HEAT AND MASS TRANSFER IN MOLECULAR GASES

S. A. Savkov^a and A. A. Yushkanov^b

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The analytical solution of the problem on a nonstationary heat source in a diatomic gas has been given. The characteristics of the temperature and sound waves excited by it have been obtained. The distribution of the pressure, temperature, and concentration of the gas molecules has been considered.

Recent years have seen a growing interest in studying nonstationary heat and mass transfer in molecular gases [1]. Consideration of phenomena of such a kind is necessary, in particular, for analysis of the thermal effects of interaction between laser radiation and a substance [2], which is pressing in investigating the self- and defocusing of a laser beam in absorbing media [3–5], especially in the case of a variable (with time) radiation power. Of independent interest is also the behavior of a gas in the vicinity of a heated solid wall [6], including the formation and propagation of thermoacoustic waves [7]. However, in theoretical description of the processes enumerated, the authors restrict their consideration to the dynamics of a continuum, whereas at distances comparable to the mean free path of molecules the state of the gas is determined by the kinetic equation [8]

$$\frac{\partial f}{\partial t} + \mathbf{V}\nabla f = J[f] + G. \quad (1)$$

The procedure of solution of the analogous problem for an atomic gas has been presented in sufficient detail in [9]. The present paper is devoted to a diatomic gas.

Restricting ourselves to allowance for effects linear in the temperature and concentration difference, we represent the solution of (1) in the form

$$f = f_0(1 + \varphi).$$

Passing to dimensionless variables, we obtain

$$\frac{\partial \varphi}{\partial t} + \mathbf{C}\nabla \varphi = I[\varphi] + W(t, \mathbf{r})g(C, \gamma, \mathbf{r}).$$

Following [10], we take

$$I[\varphi] = \sum_{i=1}^3 p_i M_i - \varphi,$$

where

$$M_i = 2\pi^{-3/2} \int p_i \varphi \exp(-C^2 - \gamma^2) \gamma d\gamma d^3 C, \quad p_1 = 1, \quad p_2 = \sqrt{\frac{2}{5}} \left(C^2 + \gamma^2 - \frac{5}{2} \right),$$

$$p_3 = \sqrt{2} C_r.$$

The function $W(t, \mathbf{r})$ can be represented in the form of the Fourier integral

^aOrel State University, 95 Komsomol'skaya Str., Orel, 302015, Russia; ^bMoscow State Regional University, 10a Radio Str., Moscow, 107005, Russia. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 77, No. 2, pp. 153–159, March–April, 2004. Original article submitted September 2, 2002; revision submitted September 30, 2003.

$$W(t, \mathbf{r}) = \int_{-\infty}^{+\infty} W_{\omega}(\mathbf{r}) \exp(i\omega t) d\omega, \quad \text{here} \quad W_{\omega}(\mathbf{r}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} W(t, \mathbf{r}) \exp(-i\omega t) dt,$$

which enables us to consider the distribution of macroscopic characteristics of the gas as a superposition of the corresponding quantities created by individual harmonics. The heat-release region itself can be represented as a set of point sources. Thus, the problem is reduced to solution of the equation

$$\frac{\partial \varphi}{\partial t} + \mathbf{C}\nabla\varphi = I[\varphi] + \exp(i\omega t) S(C, \gamma) \delta(r),$$

determining the state of the gas around an isotropic point source located at the origin of the coordinate system.

In the case of the heat source we obtain

$$S = S_h = \frac{2}{5} (C^2 + \gamma^2) - 1.$$

In separate excitation of the translational and rotational degrees of freedom, we have

$$S = S_v = \frac{2}{3} C^2 - 1 \quad \text{and} \quad S = S_{\omega} = \gamma^2 - 1$$

respectively.

For the sake of completeness we also consider the particle source:

$$S = S_p = 1.$$

Following considerations analogous to those presented in [9], we obtain the following expression determining the dependence of the macroscopic parameters of the gas on the distance to the source:

$$M_s^m(t, \sigma, r) = \frac{\sigma + 1}{2\pi^{3/2} r} \sum_{\alpha} \frac{a_{\alpha}^s a_{\alpha}^m}{\eta_{\alpha} N_{\alpha}} \exp(\sigma t - (\sigma + 1) r / \eta_{\alpha}) +$$

$$+ \frac{(\sigma + 1)^3}{2\pi^{3/2} r} \int_0^{\infty} \frac{N_{11} b_1^s b_1^m + N_{22} b_2^s b_2^m - N_{12} (b_1^s b_2^m + b_2^s b_1^m)}{\eta N_0} \exp(\sigma t - (\sigma + 1) r / \eta) d\eta \quad (s, m = 1, 2, 3, 4),$$

$$\sigma = i\omega, \quad a_{\alpha}^1 = \frac{2}{5} \frac{\eta_{\alpha}^2 \sigma^2}{\sigma + 1} - \frac{\sigma + 1}{5},$$

$$a_{\alpha}^2 = \frac{10(\sigma + 1) + \lambda_c(\eta_{\alpha})}{30} \left(2 \left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - 1 \right), \quad a_{\alpha}^3 = \frac{\lambda_c(\eta_{\alpha})}{20} \left(1 - 2 \left(\frac{\sigma \eta}{\sigma + 1} \right)^2 \right),$$

$$a_{\alpha}^4 = \sigma + 1 + \frac{4}{5} \lambda_c(\eta_{\alpha}), \quad b_1^1 = b_2^4 = 1, \quad b_2^1 = b_1^4 = 0,$$

$$b_1^2 = \frac{5}{6}, \quad b_1^3 = \frac{5}{4}, \quad b_2^2 = \frac{1}{3} \left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - \frac{1}{6}, \quad b_2^3 = \frac{1}{4} - \frac{1}{2} \left(\frac{\sigma \eta}{\sigma + 1} \right)^2,$$

$$N_{11} = \left(\sigma + 1 + \lambda_p(\eta) + 2(\lambda_p(\eta) + 1) \frac{\sigma \eta^2}{\sigma + 1} \right)^2 + \frac{1}{8} \left(\sigma + 1 - \frac{2\eta^2 \sigma^2}{\sigma + 1} \right)^2 + \pi \eta^2 \exp(-2\eta^2) \left(1 + \frac{2\sigma \eta^2}{\sigma + 1} \right),$$

$$\begin{aligned}
N_{12} &= \left(\sigma + 1 + \lambda_p(\eta) + 2(\lambda_p(\eta) + 1) \frac{\sigma \eta^2}{\sigma + 1} \right)^2 \left(\lambda_p(\eta) \left(\eta^2 - \frac{1}{2} \right) + \eta^2 \right) + \\
&+ \frac{1}{8} \left(\sigma + 1 - \frac{2\eta^2 \sigma^2}{\sigma + 1} \right) (5(\sigma + 1) + 4\lambda_p(\eta)) + \pi \eta^2 \exp(-2\eta^2) \left(\eta^2 - \frac{1}{2} \right) \left(1 + \frac{2\sigma \eta^2}{\sigma + 1} \right), \\
N_{22} &= \left(\lambda_p(\eta) \left(\eta^2 - \frac{1}{2} \right) + \eta^2 \right)^2 + \frac{1}{8} (5(\sigma + 1) + 4\lambda_p(\eta))^2 + \pi \eta^2 \exp(-2\eta^2) \left(\left(\eta^2 - \frac{1}{2} \right)^2 + 2 \right), \\
N_0 &= \eta \exp(\eta^2) (N_{11} N_{22} - N_{12}^2).
\end{aligned}$$

Summation in the first term of (2) should be only over those η_α for which $\text{Re}((\sigma + 1)/\eta_\alpha) > 0$.

Because of the strong cumbersomeness, it is more convenient to compute the values of N_α by direct numerical integration:

$$N_\alpha = \frac{1}{\pi} \frac{\eta^2}{(\sigma + 1)^2} \int_{-\infty}^{+\infty} \left((\Delta_{11}\Lambda_{22} - \Delta_{12}\Lambda_{21})^2 + (\Delta_{21}\Lambda_{22} - \Delta_{22}\Lambda_{21})^2 \right) \frac{\exp(-\mu^2)}{(\eta - \mu)^2} d\mu,$$

where Δ_{ij} and Λ_{ij} are the components of the matrices

$$\Delta = \begin{bmatrix} 1 + 2 \frac{\mu \sigma \eta}{\sigma + 1} + \frac{2}{5} \left(\mu^2 - \frac{1}{2} \right) \left(\left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - \frac{1}{2} \right) & \frac{2}{5} \sqrt{2} \left(\mu^2 - \frac{1}{2} \right) \\ \frac{2}{5} \sqrt{2} \left(\left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - \frac{1}{2} \right) & \frac{4}{5} \end{bmatrix}$$

and

$$\Lambda(\sigma, \eta) = \begin{bmatrix} \sigma + 1 + \lambda_c(\eta) + 2 \frac{\sigma \eta^2}{\sigma + 1} (\lambda_c(\eta) + 1) + \lambda_1(\eta) \left(\left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - \frac{1}{2} \right) & \sqrt{2} \lambda_1(\eta) \\ \frac{2}{5} \sqrt{2} \lambda_c(\eta) \left(\left(\frac{\sigma \eta}{\sigma + 1} \right)^2 - \frac{1}{2} \right) & \sigma + 1 + \frac{4}{5} \lambda_c(\eta) \end{bmatrix},$$

computed for $\eta = \eta_\alpha$,

$$\lambda_c(\eta) = \eta \pi^{-1/2} \int_{-\infty}^{+\infty} \exp(-\mu^2) \frac{d\mu}{\mu - \eta}, \quad \lambda_1(\eta) = \frac{2}{5} \left(\lambda_c(\eta) \left(\eta^2 - \frac{1}{2} \right) + \eta^2 \right),$$

$$\lambda_p(\eta) = -2\eta \exp(-\eta^2) \int_0^\eta \exp(\mu^2) d\mu.$$

The values of η_α are found from the equation

$$D(\eta) = \det \Lambda(\sigma, \eta) = 0. \tag{3}$$

As has been shown in [9], the dispersion function $D(\eta)$ is an even piecewise analytical function in a complex plane with a section along the real axis and can be represented in the form

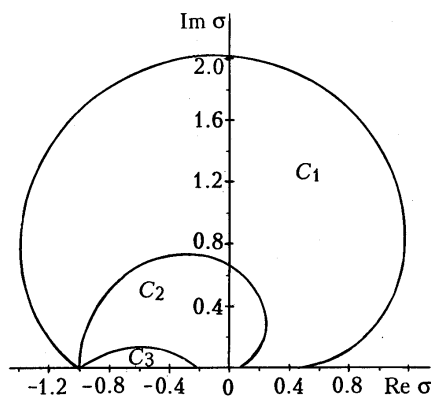


Fig. 1. Region of σ values in which the dispersion equation has two (C_1), four (C_2), and six (C_3) roots of the discrete spectrum.

$$D(z) = A(z+i)^{-2\kappa} X^\pm(z) \prod_{\alpha=1}^{\kappa} (\eta_\alpha^2 - z^2) \quad \text{at } \pm \text{Im } z > 0,$$

where

$$X^\pm(z) = \left(\frac{z+i}{z \pm i} \right)^{2\kappa} \exp(\Gamma(z)); \quad \Gamma(z) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \ln \left(\frac{\mu+i}{\mu-i} \right)^{2\kappa} G(\mu) \frac{d\mu}{\mu-z};$$

$$G(x) = \lim_{y \rightarrow +\infty} \frac{D(x+iy)}{D(x-iy)}; \quad \kappa = \frac{\Delta \arg G(x)}{2\pi};$$

$\Delta \arg G(x)$ is the increment in the argument of the function G as x changes from $-\infty$ to $+\infty$.

Thus, to determine η_α it is sufficient to compute D and X^\pm for arbitrary $(\kappa+1)$ th values of z and to solve the resulting system of equations for the above quantities and the constant A . The values of $\pm \eta_\alpha$ found in such a manner are the sought roots of (3).

Figure 1 shows the regions C_1 , C_2 , and C_3 of variation of the parameter σ in which the dispersion equation (3) has two, four, and six roots respectively. For the negative half-space of $\text{Im } \sigma$ the picture has a form symmetric about the real axis.

Figure 2a and b gives the values of the relative difference of the temperature and concentration of the gas molecules, which are created by a point heat source. The distribution of the remaining moments has an analogous form. The exception is $\Delta T_p^\omega = \Delta N_\omega$, whose plots are presented in Fig. 2c. The symmetry of M_s^m relative to the rearrangement of the indices is noteworthy.

In the immediate vicinity of the source, i.e., when $r \ll |i\omega+1|$, the main contribution to expression (2) is made by the solutions of a continuous spectrum. The value of the integrals is determined by the low values of ω , for which

$$\lambda_p = 0, \quad \int_0^\infty \frac{\exp(-(i\omega+1)r/\eta)}{\eta^2} d\eta = \frac{1}{r(i\omega+1)},$$

$$N_{11} = \frac{9}{8}(i\omega+1)^2, \quad N_{12} = \frac{5}{8}(i\omega+1)^2, \quad N_{22} = \frac{25}{8}(i\omega+1)^2, \quad N_0 = \frac{25}{8}(i\omega+1)^4 \eta.$$

Accordingly, we have

$$\Delta T_h = \frac{9}{50\pi^{3/2} r^2}, \quad \Delta T_h^\omega = \Delta T_\omega = \frac{1}{5\pi^{3/2} r^2}, \quad \Delta N_h = \Delta T_p = -\frac{1}{10\pi^{3/2} r^2},$$

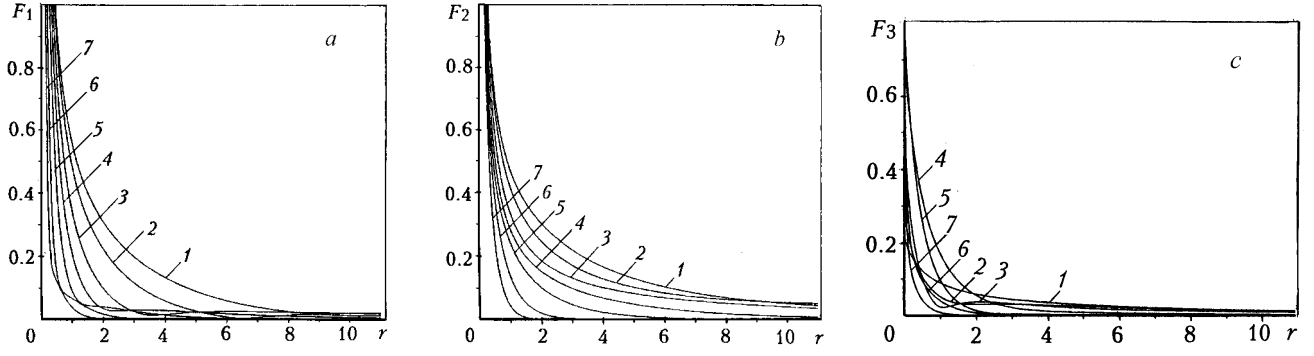


Fig. 2. Values of $F_1 = 2\pi^{3/2}r|\Delta T_h|$ (a), $F_2 = 2\pi^{3/2}r|\Delta N_h|$ (b), and $F_3 = 2\pi^{3/2}r|\Delta N_\omega|$ (c) vs. distance to the source for fixed values of ω : 1) $\omega = 0.1$; 2) 0.2; 3) 0.5; 4) 1; 5) 2; 6) 5; 7) 10.

$$\Delta T_v = \Delta T_v^y = \Delta T_v^\omega = \Delta T_\omega^y = -\Delta N_v = -\Delta T_p^y = \frac{1}{6\pi^{3/2}r^2}.$$

Thus, the distribution of most of the moments in the immediate vicinity of the source is independent of ω and is prescribed only by its instantaneous power. The values of $\Delta T_p^\omega = \Delta N_\omega$ in the case in question are in proportion to $1/r$ and are determined by the consequents in the expansion in η .

With distance from the source the second term in (2) is damped more rapidly than the first term. Therefore, in the limit $r \rightarrow \infty$, the value of M_s^m is prescribed by the solutions (if any) of a discrete spectrum. The rate of damping of these solutions depends on the value of η_α . As a result, the one which makes a larger contribution to the distribution of macroscopic parameters in the immediate vicinity of the source is damped more rapidly.

In the case of low frequencies ($\omega \rightarrow 0$) the roots of the dispersion equation are determined by the relations

$$\eta_{\pm 1} = \pm \sqrt{\frac{7}{10}} \frac{i}{\omega}, \quad \eta_{\pm 2} = \pm \frac{1-i}{2\sqrt{\omega}}.$$

Accordingly, we obtained

$$N_{\pm 1} \sqrt{\pi} \eta_{\pm 1} = \pm \frac{49}{625} \frac{i}{\omega}, \quad N_{\pm 2} \sqrt{\pi} \eta_{\pm 2} = \pm \frac{7}{50},$$

$$a_{\pm 1}^1 = \frac{2}{25}, \quad a_{\pm 1}^4 = a_{\pm 2}^4 = -a_{\pm 2}^1 = \frac{1}{5}, \quad a_{\pm 1}^2 = \frac{3}{25}, \quad a_{\pm 2}^2 = -\frac{3}{10}, \quad a_{\pm 1}^3 = \frac{1}{50}, \quad a_{\pm 2}^3 = -\frac{1}{20}.$$

As a result we have

$$\begin{aligned} \Delta T_h &= \Delta T_h^{\text{as}} + I_{11}, \quad \Delta T_h^y = \Delta T_v = \Delta T_h^{\text{as}} + \frac{5I_{11} + I_{12}}{6}, \quad \Delta T_h^\omega = \Delta T_\omega = \Delta T_h^{\text{as}} + \frac{5I_{11} - I_{12}}{4}, \\ \Delta T_p &= \Delta T_p^{\text{as}} - I_{12}, \quad \Delta T_v^y = \Delta T_h^{\text{as}} + \frac{25I_{11} + 10I_{12} + I_{22}}{36}, \quad \Delta T_\omega^y = \Delta T_v^\omega = \Delta T_h^{\text{as}} + \frac{25I_{11} - I_{22}}{24}, \\ \Delta T_\omega^\omega &= \Delta T_h^{\text{as}} + \frac{25I_{11} - 10I_{12} + I_{22}}{16}, \quad \Delta T_p^y = \Delta T_p^{\text{as}} - \frac{5I_{12} + I_{22}}{6}, \quad \Delta T_p^\omega = \Delta T_p^{\text{as}} - \frac{5I_{12} - I_{22}}{4}, \\ \Delta N_h &= \Delta N_h^{\text{as}} - I_{12}, \quad \Delta N_v = \Delta N_h^{\text{as}} - \frac{5I_{12} + I_{22}}{6}, \quad \Delta N_\omega = \Delta N_h^{\text{as}} - \frac{5I_{12} - I_{12}}{4}, \end{aligned}$$

$$\Delta N_p = \Delta N_p^{\text{as}} + I_{22},$$

TABLE 1. Parameters of the Sound Wave

ω	η_1	A_1	k_1	d_1	δ_1
0.001	1.43427-836.666 <i>i</i>	0.00090	0.00120	0.00000	0.49941π
0.01	1.43417-83.6659 <i>i</i>	0.00897	0.01195	0.00009	0.49409π
0.1	1.42524-8.36315 <i>i</i>	0.08751	0.11818	0.00818	0.44377π
0.2	1.40559-4.16599 <i>i</i>	0.16596	0.23005	0.02961	0.39926π
0.3	1.38494-2.74859 <i>i</i>	0.23611	0.33402	0.05915	0.36603π
0.4	1.36685-2.02388 <i>i</i>	0.30138	0.43100	0.09344	0.34028π
0.5	1.35175-1.57661 <i>i</i>	0.36413	0.52226	0.13064	0.31936π
0.6	1.33921-1.26895 <i>i</i>	0.42575	0.60889	0.16977	0.30171π
0.666	1.33208-1.11224 <i>i</i>	0.46620	0.66392	0.19635	0.29140π
0.667	1.33197-1.11008 <i>i</i>	0.46681	0.66474	0.19676	0.29125π
0.7	1.32870-1.04188 <i>i</i>	0.48703	0.69169	0.21024	0.28643π
0.8	1.31980-0.86585 <i>i</i>	0.54844	0.77128	0.25170	0.27293π
0.9	1.31215-0.72435 <i>i</i>	0.61029	0.84814	0.29391	0.26084π
1.0	1.30550-0.60743 <i>i</i>	0.67276	0.92265	0.33670	0.24988π
1.1	1.29965-0.50869 <i>i</i>	0.73597	0.99510	0.37995	0.23987π
1.2	1.29446-0.42383 <i>i</i>	0.80000	1.06572	0.42359	0.23065π
1.3	1.28981-0.34983 <i>i</i>	0.86492	1.13471	0.46754	0.22210π
1.4	1.28562-0.28454 <i>i</i>	0.93075	1.20223	0.51175	0.21415π
1.5	1.28180-0.22633 <i>i</i>	0.99753	1.26843	0.55618	0.20671π
1.6	1.27831-0.17398 <i>i</i>	1.06527	1.33342	0.60080	0.19972π
1.7	1.27511-0.12654 <i>i</i>	1.13399	1.39729	0.64558	0.19315π
1.8	1.27214-0.08328 <i>i</i>	1.20368	1.46014	0.69050	0.18693π
1.9	1.26939-0.04358 <i>i</i>	1.27435	1.52204	0.73553	0.18104π
2.0	1.26682-0.00697 <i>i</i>	1.34600	1.58305	0.78067	0.17545π
2.019	1.26635-0.00033 <i>i</i>	1.35972	1.59455	0.78926	0.17441π

TABLE 2. Parameters of the Temperature Wave

ω	η_2	A_2	k_2	d_2	δ_2
0.001	15.8395-15.7831 <i>i</i>	0.00090	0.03160	0.03165	-0.50005π
0.01	5.08692-4.90869 <i>i</i>	0.00898	0.09925	0.10081	-0.50042π
0.1	1.79078-1.28036 <i>i</i>	0.09038	0.30115	0.34310	-0.49615π
0.2	1.35867-0.71278 <i>i</i>	0.17690	0.41822	0.51661	-0.49241π
0.3	1.16862-0.44118 <i>i</i>	0.25908	0.50744	0.66414	-0.49464π
0.4	1.05611-0.26967 <i>i</i>	0.33931	0.58254	0.79813	-0.49980π
0.5	0.97977-0.14685 <i>i</i>	0.41908	0.64873	0.92342	-0.50613π
0.6	0.92365-0.05232 <i>i</i>	0.49924	0.70865	1.04251	-0.51282π
0.666	0.89384-0.00036 <i>i</i>	0.55259	0.74555	1.11847	-0.51725π

which coincides with the distribution of M_S^m from a stationary source. Here

$$I_{ij} = \frac{1}{2\pi^{3/2} r_0} \int_0^\infty \frac{N_{ij}}{\eta N_0} \exp(-r/\eta) d\eta .$$

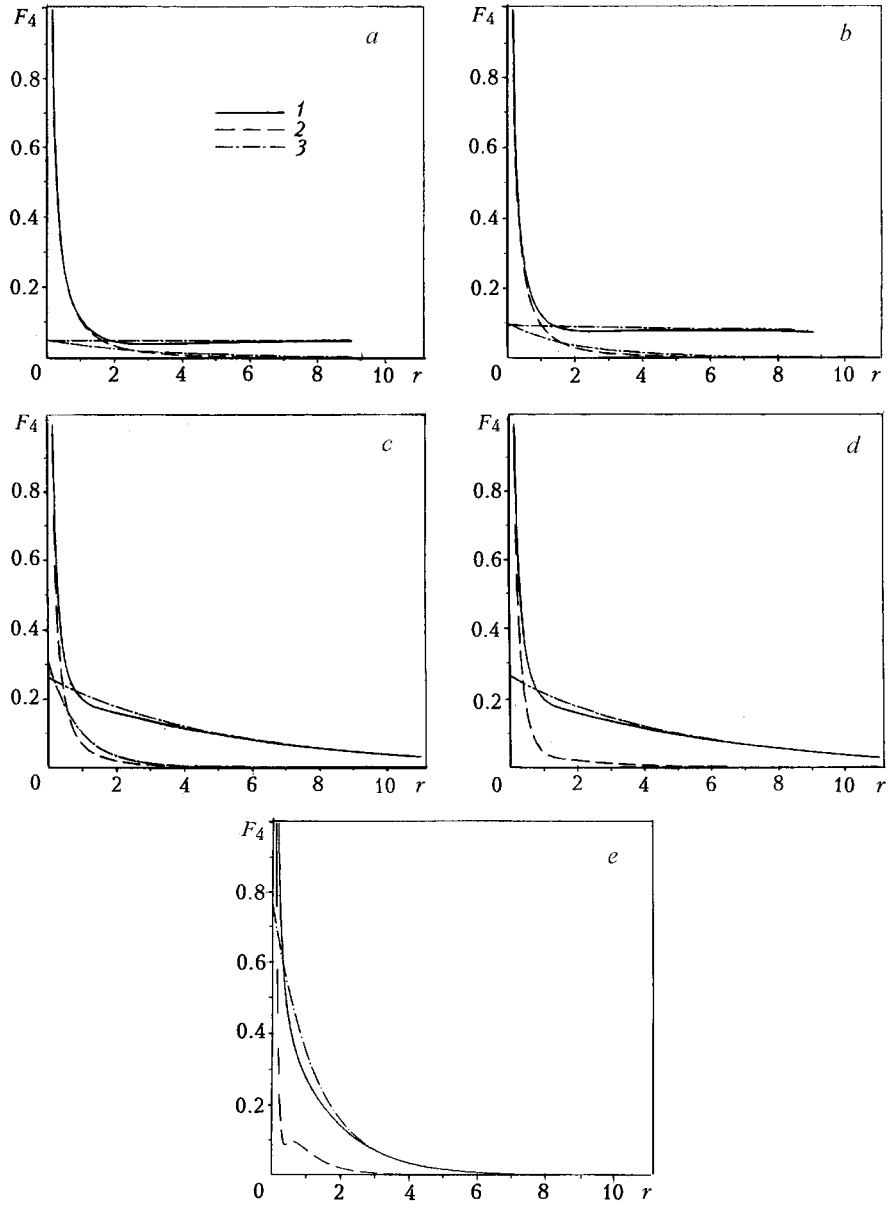


Fig. 3. Values of $F_4 = 2\pi^{3/2}r|\Delta P_h|$ vs. distance to the source for $\omega = 0.1$ (a), 0.2 (b), 0.666 (c), 0.667 (d), and 2.019 (e) (1); contribution of the solutions of the continuous (2) and discrete (3) spectra.

The functions

$$\Delta T_h^{as} = -\Delta N_h^{as} = \Delta N_p^{as} = -\Delta T_p^{as} = \frac{1}{7\pi r}$$

describe the gasdynamic distribution of the temperature and concentration of the gas molecules which is created by unit-power sources of heat and particles. It is noteworthy that the above terms are determined by the solution of the discrete spectrum, which corresponds to the eigenvalue of η_2 .

Of independent interest is to consider the relative difference of the gas pressure

$$\Delta P = \frac{P - P_0}{P_0} = \Delta T + \Delta N,$$

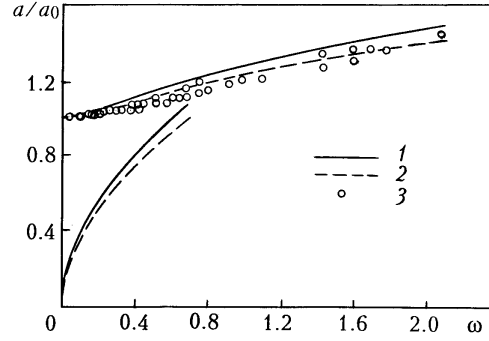


Fig. 4. Phase velocity of the sound (upper group of curves) and temperature (lower group) waves vs. frequency: 1) in the diatomic gas; 2) in the atomic gas; 3) results of the experiments [11] on measurement of the velocity of sound in inert gases.

which can be written in the form of a superposition of spherical waves

$$\Delta P = \frac{1}{2\pi r} \sum_{\alpha=1}^{\kappa} A_{\alpha} \exp(i(\omega t - k_{\alpha} r) - d_{\alpha} r + \delta_{\alpha}) + \frac{1}{2\pi r} \int_0^{\infty} A_c \exp(i(\omega t - k_c r) - d_c r + \delta_c) \delta \eta.$$

The quantities $k_{\alpha} = \text{Im}((i\omega + 1)/\eta_{\alpha})$ and $d_{\alpha} = \text{Re}((i\omega + 1)/\eta_{\alpha})$ are determined solely by the structure of the dispersion equation and are independent of the character of the source. The values of the parameters indicated are presented in Tables 1 and 2; the tables also give A_{α} and δ_{α} for $S = S_h$.

The phase rate of the above wave processes is prescribed by the relation $a_{\alpha} = \omega/k_{\alpha}$, which yields $a_1 = a_0 = \sqrt{5/6}$ and $a_2 = \sqrt{\omega}$ in the limit $\omega \rightarrow 0$. Passing to dimensional quantities, we find $a_0 = \sqrt{5kT/3m}$, which coincides with the velocity of sound computed within the framework of the dynamics of continua. The value of $a_a = \sqrt{2\omega\chi}$ corresponds to the velocity of propagation of a temperature wave.

The pressure distributions are plotted in Fig. 3.

As is clear from the plots and tables presented, the amplitude of each wave increases with frequency; the rate of its damping also increases, tending to a wave corresponding to the solutions of the continuous spectrum, as ω approaches the boundaries of the regions of C_{α} .

Figure 4 gives the phase velocity of the sound and temperature waves as a function of the frequency.

It is noteworthy that the sound and temperature waves have opposite values of the initial phase in the limit of low frequencies. The former is one quarter of a period ahead of the source, whereas the latter lags behind it by the same value.

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NOTATION

A_{α} and a_{α} , amplitude and phase rate of the wave processes corresponding to the solutions of the discrete spectrum; A_c and a_c , the same for the solutions of the continuous spectrum; $C = \mathbf{V}\sqrt{m/2kT_0}$, dimensionless value of the velocity of translational thermal motion of gas molecules; d_{α} and d_c , logarithmic decrement of the wave processes corresponding to the discrete and continuous spectra of solutions; f , distribution function; $f_0 = n_0 \left(\frac{m}{2\pi kT_0} \right)^{3/2} \frac{J}{kT_0} \exp(-C^2 - \gamma^2)$, equilibrium (Maxwell) distribution; G , phase distribution density of the sources; g , corresponding dimensionless phase density for a unit-power source; i , imaginary unit; $I[\phi]$ and $J[f]$, linearized and nonlinearized collision operators; J , moment of inertia of the gas molecules; k , Boltzmann constant; k_{α} and k_c , wave numbers corresponding

to the solutions of the discrete and continuous spectra; $M_s^1 = \Delta T_s = \frac{T - T_0}{T_0} = \frac{4}{5}\pi^{-3/2} \int \left(C^2 + \gamma^2 - \frac{5}{2} \right) \varphi_s \exp(-C^2 - \gamma^2) \gamma d\gamma d^3 C$; $M_s^2 = \Delta T_s^v = \frac{T_v - T_0}{T_0} = \frac{4}{3}\pi^{-3/2} \int \left(C^2 - \frac{3}{2} \right) \varphi_s \exp(-C^2 - \gamma^2) \gamma d\gamma d^3 C$; $M_s^3 = \Delta T_s^\omega = \frac{T_\omega - T_0}{T_0} = 2\pi^{-3/2} \int (\gamma^2 - 1) \varphi_s \exp(-C^2 - \gamma^2) \gamma d\gamma d^3 C$; $M_s^4 = \Delta N_s = \frac{n - n_0}{n_0} = 2\pi^{-3/2} \int \varphi_s \exp(-C^2 - \gamma^2) \gamma d\gamma d^3 C$; m and n , mass and concentration of the gas molecules; P , pressure; \mathbf{r} , radius vector of the point in question; S , dimensionless value of the phase density of a unit-power point source; T , temperature; t , time; \mathbf{V} , thermal velocity of translational motion of the gas molecules; W , source-power distribution density; δ_α and δ_c , initial phase of the wave processes corresponding to the solutions of the discrete and continuous spectra; η_α , roots of the dispersion equation (3); κ , number of root pairs of the dispersion equation; $\gamma = \Omega\sqrt{J/2kT_0}$, dimensionless value of the velocity of rotational motion of the gas molecules; φ , correction to the equilibrium distribution function; Ω , thermal velocity of rotational motion of the gas molecules; χ , thermal diffusivity of the gas; ω , cyclic frequency of change of the source power; $\sigma = i\omega$; Δ , relative difference of the corresponding quantity. Subscripts and superscripts: as, asymptotic, i.e., realized at a fairly large distance from the source, distribution of the gas parameters; i and j , i th and j th quantities; c in the function $\lambda_c(\eta)$, computation of the integral of a complex number, in the remaining cases, belonging to the continuous spectrum of solutions of the kinetic equation; h , heat; m , prescribes the form of the distribution moment; p in the function $\lambda_p(\eta)$, quantity computed in terms of the principal value of the Cauchy integral, in the remaining cases corresponds to the particle source; s , source, prescribes the character of the source and corresponds to $s = 1 - S_h$, $2 - S_v$, $3 - S_\omega$, and $4 - S_p$; v , translational degrees of freedom of the gas molecules; α , α th quantity; ω , rotational degree of freedom of the gas molecules; 0 , unperturbed values of the parameters of the gas.

REFERENCES

1. V. M. Zhdanov and M. Ya. Alievskii, *Processes of Transfer and Relaxation in Molecular Gases* [in Russian], Nauka, Moscow (1989).
2. A. I. Osipov and V. Ya. Panchenko, *Thermal Effects in Interaction between Laser Radiation and Molecular Gases* [in Russian], Izd. MGU, Moscow (1983).
3. S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Self-focusing and diffraction of light in a nonlinear medium, *Usp. Fiz. Nauk*, **93**, No. 1, 19–70 (1967).
4. A. D. Wood, M. Camac, and E. T. Gerry, Effect of 10.6- μm laser induced air chemistry on the atmospheric refractive index, *Appl. Opt.*, **10**, No. 8, 1877–1882 (1971).
5. E. L. Breig, Limitation on the atmospheric thermal effects for high-power CO laser beams, *JOSA*, **62**, No. 4, 518–525 (1972).
6. D. C. Wadsworth, D. A. Erwin, and E. P. Muntz, Transient motion of a confined rarefied gas due to wall heat or cooling, *J. Fluid Mech.*, **248**, 219–235 (1993).
7. Yu. Huang and H. H. Bau, Thermoacoustic waves in a semi-infinite medium, *Int. J. Heat Mass Transfer*, **38**, No. 8, 1329–1345 (1995).
8. C. Cercignani, *Theory and Applications of the Boltzmann Equation* [Russian translation], Mir, Moscow (1978).
9. S. A. Savkov and A. A. Yushkanov, Analytical solution of the BGK model of the nonstationary Boltzmann equation, *Teor. Mat. Fiz.*, **113**, No. 1, 139–148 (1997).
10. S. A. Savkov and A. A. Yushkanov, Accounting for energy accommodation in calculating the heat flux from a spherical particle in a diatomic gas, *Inzh.-Fiz. Zh.*, **75**, No. 5, 149–154 (2002).
11. M. Greenspan, Propagation of sound in five monatomic gases, *J. Acoust. Soc. Am.*, **28**, No. 4, 644–648 (1956).